The Interdependency of Random Rates of Return

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Stock Sample

Twitter (TWTR)	Netflix (NFLX)	Canada Goose (GOOS)	General Electric (GE)	Macy's (M)	RE/MAX (RMAX)
Tesla (TSLA)	Ford (F)	Goldman Sachs (GS)	NIO Limited (NIO)	McDonalds (MCD)	Nucor (NUE)
Microsoft (MSFT)	Pfizer (PFE)	JP Morgan Chase (JPM)	Exxon Mobil (XOM)	Chipotle (CMG)	D.R. Horton (DHI)
Facebook (FB)	Amazon (AMZN)	Caterpillar (CAT)	Walgreens (WBA)	Starbucks (SBUX)	Univar (URL)
Apple (APPL)	Pepsi (PEP)	Johnson + Johnson (JNJ)	Best Buy (BBY)	CVS (CVS)	American Realty (ARL)

years sampled 2020, 2017, 2008, 2005 based on volatility

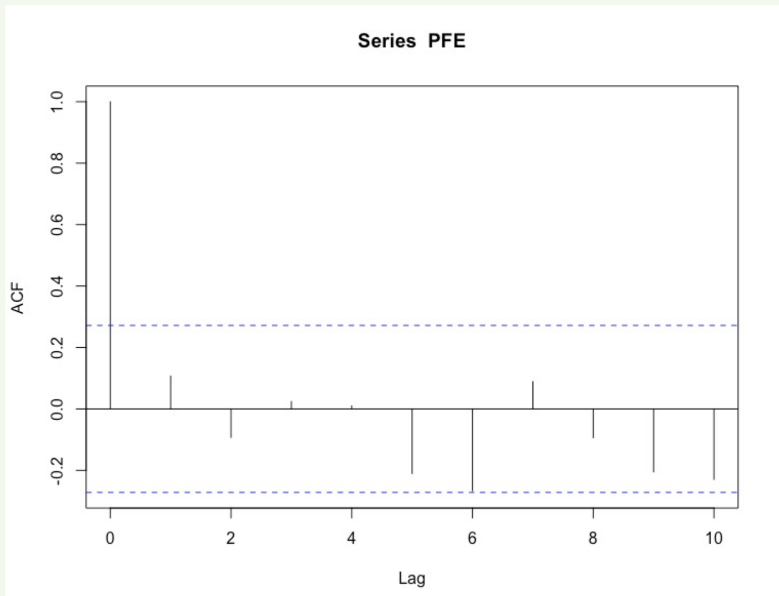
Autocorrelation Functions & Partial Autocorrelation Functions

- Traditionally used to identify ARIMA series
- Used here to identify strong correlation patterns between time lags

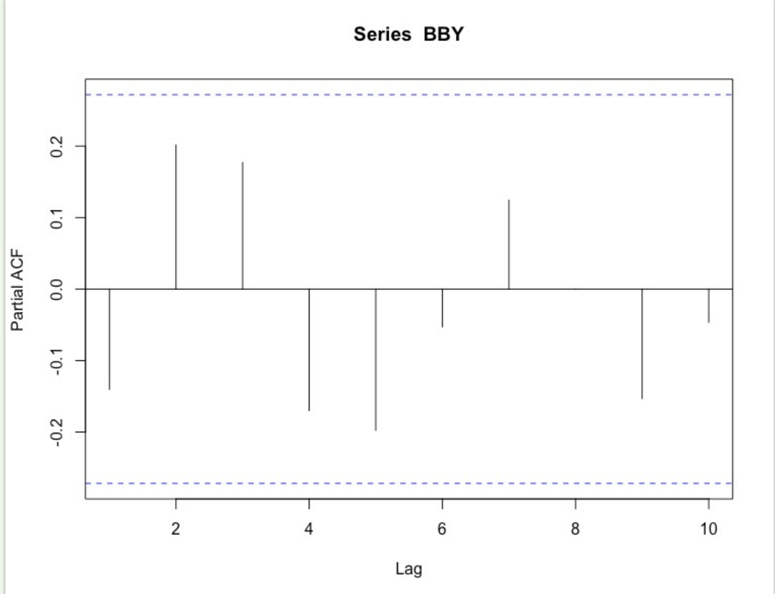
Autocorrelation Calculation

```
stockdiffs = Table[logstock[[i + 1]] - logstock[[i]], {i, 1, Length[logstock] - 1}];
acflist = Table[, {i, 0, 10}];
For[i = 1, i ≤ 11, i++,
 a = 1/Length[logstock] * (Sum[(stockdiffs[[t]] - Mean[stockdiffs]) * (stockdiffs[[t + (i - 1)]] - Mean[stockdiffs]), {t, 1, Length[stockdiffs] - (i - 1)}]);
 b = Sum[((stockdiffs[[t]] - Mean[stockdiffs])^2), {t, 1, Length[stockdiffs]}] / Length[stockdiffs];
 acflist[[i]] = a / b;
```

Autocorrelation Function Graph Example Pfizer (PFE) 2020



Partial Autocorrelation Function Graph Example Best Buy (BBY) 2020



Independence Testing

Chi-Square Independence Test

Fail to Reject
Univar (UNVR) 2017

```
Independence test:
Value of chi-square test statistic: 6.78517
Critical value: 16.919
Fail to reject HO at level 0.05
p-value of test: 0.659475
```

Reject
Ford (F) 2008

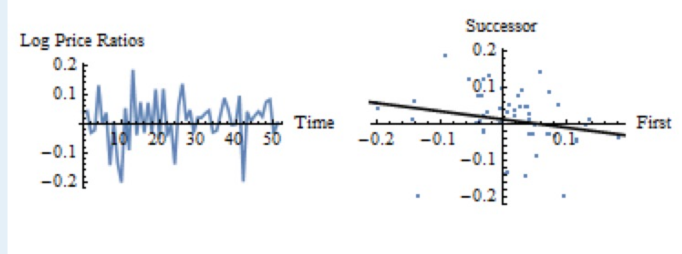
```
Independence test:
Value of chi-square test statistic: 17.6573
Critical value: 16.919
Reject H0 at level 0.05
p-value of test: 0.0393636
```

	.05	.10	Reject
Chi Square I-Test	3	12	92

Correlation Test

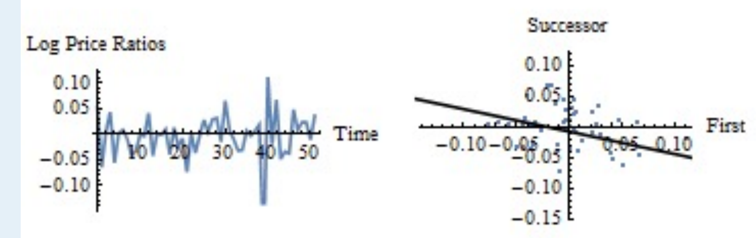
Fail to Reject
Twitter (TWTR) 2020

```
Correlation Coef: -0.223509
Student T Distribution p-value: 0.114882
```



Reject
Pfizer (PFE) 2008

```
Correlation Coef: -0.368563
Student T Distribution p-value: 0.00778591
```



	.05	.10	Reject
Chi Square I-Test	9	14	90

Runs Test

Fail to Reject
Tesla (TSLA) 2020

```
Two-sided runs test
Rejection region: R <= 19 or R >= 35
Exact level of test of nominal level 0.05 : 0.0341111
Value of test statistic: R = 27
Fail to reject H0 at this level.
p-value of test = 0.55048
```

Reject
Goldman (GS) 2008

```
Two-sided runs test
Rejection region: R <= 19 or R >= 35
Exact level of test of nominal level 0.05 : 0.0329217
Value of test statistic: R = 35
H0 rejected at this level.
p-value of test = 0.0122687
```

	.05	.10	Reject
Chi Square I-Test	7	14	90

Overview

We are testing the independence of random rates of return for randomly selected stocks throughout different time periods. We want to know if stocks truly act independently because of the underlying assumption in Geometric Brownian Motion, used in the Black-Scholes Method for European Style Options Pricing.

Conclusion

	.05	.10	Fail To Reject	Total
2020	5	13	72	90
2017	5	10	72	87
2008	6	10	53	69
2005	3	7	56	66
Total	19	40	253	312

$$\frac{19 \ tests \ failed \ to \ reject \ at \ .05}{312 \ total \ tests} \approx .06$$

Since we are testing at a .05 confidence interval, we can assume that the 19 tests that were rejected are attributed to random chance during testing. Therefore, we can confidently conclude that the random rates of return act independently.