Review of previously covered proof techniques:

1) Direct Proof: Show that P $\Rightarrow$Q.
2) Proof by Contrapositive: Show that $\sim$Q $\Rightarrow\sim$P.
3) Proof by contradiction: Show that P $\wedge$ $\sim$Q leads to a contradiction.

All of these are essentially proving that P $\Rightarrow$Q. However, there are several variations on the P $\Rightarrow$ Q theme for which the following seven strategies are useful.

1) Proving P$\Leftrightarrow$Q
We know that P$\Leftrightarrow$Q is logically equivalent to (P $\Rightarrow$Q) $\wedge$ (Q $\Rightarrow$P), so we can prove P$\Leftrightarrow$Q by showing:
i) P $\Rightarrow$Q
ii) Q $\Rightarrow$P

2) Proving P$\Leftrightarrow$Q$\Leftrightarrow$R
This statement is logically equivalent to (P $\Rightarrow$Q)$\Leftrightarrow$(Q $\Rightarrow$R)$\Leftrightarrow$(R $\Rightarrow$P). So prove:
i) P $\Rightarrow$Q
ii) Q $\Rightarrow$R
iii) R $\Rightarrow$P

3) Proving P $\Rightarrow$(Q $\wedge$ R)
P $\Rightarrow$(Q $\wedge$ R) is logically equivalent to (P $\Rightarrow$Q) $\wedge$ (P $\Rightarrow$R) (By distribution), so to prove this we can use direct proof to show:
i) P $\Rightarrow$Q
ii) P $\Rightarrow$R

4) Proving P $\Rightarrow$(Q $\vee$ R).
This Statement is logically equivalent to (P $\wedge$ $\sim$Q) $\Rightarrow$R. Thus, we assume P and $\sim$Q and show that this assumption leads to R.

The following set of proof strategies are useful for proving quantitative statements such as $\forall$xP(x), $\exists$xP(x), or $\exists$!xP(x).

5) Existential proofs $\exists$xP(x)
General strategy: Construct, or find the x such that P(x) is true.

Example: The Extreme Value Theorem (EVT), which states that "If $f(x)$ is continuous over the interval [a,b] then there exists c on [a,b] such that $f(c) \geq f(x)$ for all x ϵ [a,b] and there exists d ϵ [a,b] such that $f(d) \leq f(x)$ For all x ϵ [a,b]"
To prove: find c and d.

An alternate method is to assume there does not exist an x such that P(x) (i.e. $\forall$x$\sim$P(x)), and derive a contradiction.

6) Universal proofs $\forall$xP(x)
Primary Strategy: Assume x is an arbitrary element of the given universe. Show that P(x) is true for that x.

Example: $\forall$a $\lim_{x\to a}(x^2 = a^2)$ Universe is all real numbers.
To prove: Let a$\epsilon\mathbf{R}$. Show that $\lim_{x\to a}(x^2 = a^2)$ must be true for that a.

Alternate Strategy: Assume $\exists x \sim P(x)$. Show that this leads to a contradiction.

7) Uniquness proofs $\exists ! x P(x)$

General stratagy: select x and y such that P(x) and P(y) are both true. Show that for this to be the case x=y.