

Our intuition of sets contains contradictions. Our goal is to axiomatize set theory (just like Euclid axiomatized geometry) in a way that fixes the paradoxes

The Axiomatic Method

Math is a machine for producing true statements; But what is truth? In mathematics, we know what truth is. We start with axioms which are statements which are (plausibly) true and then deduce other statements from these axioms. We deduce something by proving it with a proof. A proof is a list of statements, each of which is true.

The statements are true for one of several reasons:

1. The statement is an axiom—simple assumption taken to be true (the idea of parallel lines for example)
2. The statement is a previous result or definition—(for example, the rule of associativity)
3. The statement is true by what has already been proved
4. The statement is a hypothesis—Most statements in mathematics are of the form if...then. The hypothesis (or antecedent) is the "if" portion of the statement. The "then" portion is the consequent. (for example, if $f(x)$ is differentiable at a then $f(x)$ is continuous at a .) The hypothesis is "if $f(x)$ is differentiable at a ."
5. The statement follows from a rule of logic. Logic is the valid ways in which other true statements are put together and new true statements are "cranked" out.

Example: if $4x + 3 = 7$ then $x = 1$

$4x + 3 = 7 \rightarrow$ *hypothesis*

$4x + 3 - 3 = 7 - 3$ (previous result, equality theorem)

$4x + (3 - 3) = (7 - 3)$ (previous result, associative law of addition)

$4x = 4$ (previous result, law of addition)

$4x * .25 = 4 * .25$ (previous result, equality theory)

$x(4 * .25) = (4 * .25)$ (previous result, law of associativity)

$x = 1$ (previous result, law of reciprocals)

Intro to Logic

Definition: a proposition is a statement which is capable of being true or false. (e.g. "It is cloudy today" "1+1=2") In propositions, specific well-defined terms are vital. Some non-propositions are: "How are you" and "How much area is under the curve." Logic is about building more complicated statements by starting with simple ones. So we must start with simple, undisputable statements and build up more complicated ones.

Example: P="It is cold" and Q="It is rainy"

These basic statements are called atomic statements because they can not be broken down any further.

There are five main ways of building more complicated statements from atomic statements. They are: negation, conjunction, disjunction, implication, and biconditional.

1. Negation (Not): Notation- If P is a not proposition then $\neg P$ is the negation of P (or "not P"). If P is true then $\neg P$ must be false. If P is false then $\neg P$ must be true.

P	$\neg P$
T	F
F	T

2. Conjunction (And)- If P and Q are both propositions then the conjunction of P and Q is the statement "P and Q" which is notated as $P \wedge Q$.

P	Q	$(P \wedge Q)$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction (Or)-If P and Q are propositions then the disjunction of P and Q is the statement "P or Q" which is notated by $P \vee Q$.

P	Q	$(P \vee Q)$
T	T	T
T	F	T
F	T	T
F	F	F

4. Implication (Implies)-If P and Q are propositions then the implication of Q from P is the statement "P implies Q" which is notated by $P \Rightarrow Q$.

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

5. Biconditional (if and only if)-If P and Q are propositions then the biconditional of P and Q is the statement "P if and only if Q" which is notated by $P \Leftrightarrow Q$.

P	Q	$(P \Leftrightarrow Q)$
T	T	T
T	F	F
F	T	F
F	F	T

Propositions in this form are called a logical equivalence. Our intuition about these equivalences may be incorrect though.

Example: If you are not taking flak, you are not over the target. I am taking flak, so I must be over the target.

We can test this statement with a truth table.

Not P implies not Q is the first sentence, and P implies Q is the second sentence. So we want to see if not P implies not Q if and only if P implies Q is a true statement.

P	Q	$(\text{not}P)$	$(\text{not}Q)$	$(\text{not}P \Rightarrow \text{not}Q)$	$(P \Rightarrow Q)$	$(\text{not}P \Rightarrow \text{not}Q \Leftrightarrow P \Rightarrow Q)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

so this statement would only be true some of the time.